

2101001102050001
EXAMINATION FEBRUARY-MARCH 2024
MASTER OF ARTS EXTERNAL PART-2
MATHEMATICS -5001
INTEGRAL TRANSFORMS (ELECTIVE GROUP - 1)-
LEVEL 5

[Time: As Per Schedule]

[Max. Marks: 100]

Instructions:

1. Fill up strictly the following details on your answer book

a. Name of the Examination : **MASTER OF ARTS EXTERNAL PART-2**

b. Name of the Subject : **MATHEMATICS-5001 INTEGRAL TRANSFORMS (ELECTIVE GROUP - 1)-LEVEL 5**

c. Subject Code No : **2101001102050001**

2. Sketch neat and labelled diagram wherever necessary.

3. Figures to the right indicate full marks of the question.

4. All questions are compulsory.

5. Follow usual notations and conventions.

Seat No:

--	--	--	--	--	--

Student's Signature

Q.1 (a) If $L[f(t)] = \bar{f}(s)$ and $L[g(t)] = \bar{g}(s)$, then prove that **7**
 $L[f(t)] * g(t) = \bar{f}(s) \cdot \bar{g}(s)$

(b) (i) If $L[f(t)] = \bar{f}(s)$, then prove that $L\left[\int_0^t f(u)du\right] = \frac{\bar{f}(s)}{s}$ **7**

(ii) Find $L[(t + 1)^2 e^t]$

(c) Find $L^{-1}\left[\frac{s}{s^4+s^2+1}\right]$ **6**

OR

(a) If $L^{-1}[\bar{f}(s)] = f(t)$, then prove that $L^{-1}[\bar{f}(as)] = \frac{1}{a} f\left(\frac{t}{a}\right)$, where $a > 0$. **7**

Find $L^{-1}\left[\log\left(\frac{s+3}{s+2}\right)\right]$

(b) Using Laplace transform prove that $\int_0^\infty t e^{-3t} \sin t dt = \frac{3}{50}$ **7**

- (c) If $L[f(t)] = \bar{f}(s)$, then in usual notations prove that 6

$$L[f^n(t)] = s^n \bar{f}(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - f^{(n-1)}(0)$$

- Q.2** (a) Use Laplace transform to solve the integral equation 7

$$f(t) = 1 + \int_0^t \sin(t-u) f(u) du$$

Also verify your solution.

- (b) Use Laplace transform to solve 7

$$x''(t) + tx'(t) - 2x(t) = 2, x(0) = x'(0) = 0$$

- (c) Use Laplace transform to solve 6

$$(D^2 + D)y(t) = t^2 + 2t, y(0) = 4, y'(0) = -2$$

OR

- (a) (i) Using Laplace transform prove that $\int_0^\infty t^3 e^{-t} \sin t dt = 0$ 7

- (ii) If $L[f(t)] = \bar{f}(s)$, then prove that $L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(x) dx$, where
 $F(x) = \bar{f}(s)$

- (b) Using convolution theorem evaluate $L^{-1}\left[\frac{1}{s^2(s^2+a^2)}\right]$. 7

- (c) Find $L^{-1}\left[\frac{3s+7}{(s^2-2s-3)}\right]$ 6

- Q.3** (a) Define finite Laplace transform. 7

Find finite Laplace transform of (i) $f(t) = t^2$

(ii) $f(t) = \sin at$

- (b) Show that finite sine transforms of $\frac{x}{\pi}$ is $\frac{(-1)^{p+1}}{P}$ 7

- (c) Define (i) Finite Fourier sine transform (ii) Finite Fourier cosine transform 6

Find finite cosine transform of function $f(x) = \frac{\pi}{3} - x + \frac{x^2}{2\pi}$.

OR

(a) Using convolution theorem evaluate $L^{-1} \left[\frac{1}{(s^2+a^2)^2} \right]$. 7

(b) Find finite cosine transform of $f(x) = \sin(nx)$ 7

(c) Find finite Sine transform of $f(x) = e^{cx}$. 6

Q.4 (a) Use Fourier transform to determine the displacement $y(x, t)$ of an infinite string given that string is initially at rest and the initial displacement is $f(x)$ for $-\infty < x < \infty$. Show that solution can be put in the form

$$y(x, t) = \frac{1}{2} [f(x + ct) + f(x - ct)].$$

(b) Solve using proper Fourier transform 7

$$\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2} \text{ subject to boundary conditions}$$

1. $u(0, t) = 0$

2. $u(x, 0) = e^{-x}$ and $u(x, t)$ is bounded.

(c) Solve the integral equation using Fourier transform 6

$$\int_0^\infty f(x) \cos(\lambda x) dx = e^{-\lambda}$$

Hence show that $\int_0^\infty \frac{\cos(\lambda x)}{1+\lambda^2} d\lambda = \frac{\pi}{2} e^{-x}$.

OR

(a) Solve following B.V.P. by sine/cosine transform $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with conditions 7

1. $u(0, t) = 0$

2. $u(x, t) = \begin{cases} 1 & ; 0 < x < 1 \\ 0 & ; x \geq 1 \end{cases}$ for $t=0$

(b) The temperature u in semi finite rod is given by, 7

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \quad 0 \leq x < \infty, \text{ subject to the conditions}$$

1. $u = 0$, when $t = 0, x > 0$

2. $\frac{\partial u}{\partial x} = \mu$ when $x=0$

3. $\frac{\partial u}{\partial t} \rightarrow 0$ as $x \rightarrow \infty$

Determine temperature.

(c) Solve the following integral equation using Fourier transform. 6

$$\int_0^{\infty} f(x) \cos(sx) dx = \begin{cases} 1-s & ; 0 \leq s \leq 1 \\ 0 & ; s \geq 1 \end{cases}$$

Q.5 (a) Evaluate Fourier cosine transform of $\frac{1}{1+x^2}$. Hence find Fourier Sine transform of $\frac{x}{1+x^2}$. 7

(b) (i) Find Fourier sine transform of x^{n-1} 7

(ii) Find Fourier cosine transform of $e^{-ax} \sin(ax)$

(c) Define Fourier transform and its inverse form. State and prove linearity property for Fourier transform. 6

OR

(a) (i) If $F_c[f(x)] = f_c(s)$ Then prove that $F_c[f(ax)] = \frac{1}{a} f_c\left(\frac{s}{a}\right)$ 7

(ii) $F[f(x) \cos(ax)] = \frac{1}{2} [f(s-a) + f(s+a)]$

(b) Find Fourier sine transform of $\frac{e^{-ax} - e^{-bx}}{x}$; $a > 0, b > 0$ 7

(c) Find Fourier transform of $f(x) = \begin{cases} x & ; 0 \leq x \leq 1 \\ 0 & ; x \geq 1 \end{cases}$ 6
